The Unitarity Triangle on the First Quadrant
and the Quark Mass Matrices
in the Nearest-Neighbor Interaction Basis

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Abstract : The unitarity triangle on the first quadrant in the $\rho - \eta$ plane is discussed in the framework of the quark mass matrices in the NNI basis. If the quark mass matrices of the up-type is the Fritzsch one and the down-type is the one proposed by Branco et al., respectively, one gets the unitarity triangle with the vertex on the first quadrant. This simple model may be a candidate of the quark mass matrix ansätze, if the allowed region of the vertex of the unitarity triangle is restricted in the first quadrant by future experiments.

One of the most important problems of flavor physics is to understand flavor mixing and fermion masses, which are free parameters in the standard model. The observed values of those mixing and masses provide us with clues of the origin of the fermion mass matrices. One of the most stringent tests of the quark mass matrices is an examination of the so called unitarity triangle of the Kobayashi-Maskawa(KM) matrix[1]. Then one needs the experimental information of the six quark masses to estimate the KM matrix elements by quark mass matrix models. The discovery of top quark[2] can lead to the precise study of the quark mass matrices. Thus, we are now in the epoch of examining the quark mass matrices in terms of KM matrix elements.

As presented by Branco, Lavoura and Mota, both up and down quark mass matrices could always be transformed to the non-hermitian matrices in the nearest-neighbor interaction (NNI) basis by a weak-basis transformation for the three and four generation cases[3,4]. In this basis, the KM matrix elements are expressed generally in terms of mass matrix parameters due to eight texture zeros. In particular, phases of the mass matrices can be easily isolated. The famous Fritzsch ansätze[5] is the special one of the NNI basis. This ansätze is viable for the $|V_{us}|$ element as follows

$$V_{us} \simeq -\sqrt{\frac{m_d}{m_s}}e^{ip} + \sqrt{\frac{m_u}{m_c}}e^{iq}, \tag{1}$$

where p and q are phase parameters.

In the ref[6], we have examined the unitarity triangle from the quark mass matrices with the generation hierarchy in the NNI basis. It is emphasized that the position of vertex of the unitarity triangle is on the second quadrant of the $\rho - \eta$ plane[7] as far as Eq.(1) holds. However, the experimentally allowed region of the triangle vertex also exists on the first quadrant as well as the second one. So, in this paper, we consider the quark mass matrices that the vertex position of

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the unitarity triangle stays on the first quadrant. In the ref.[6], it is shown that there is a possibility that the vertex of the unitarity triangle moves into the first quadrant if the large discrepancy from Fritzsch ansätze will be obtained in future experiments. We propose a simple example of the quark mass matrices which implies the unitarity triangle with the first quadrant vertex.

Let us begin with considering two typical matrices for the quark mass matrices,

$$
M_1 = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} \quad , \quad M_2 = \begin{pmatrix} 0 & D & 0 \\ D^* & 0 & E \\ 0 & F & F \end{pmatrix} \quad ,
\tag{2}
$$

where A, B, D, E and F are complex numbers, while C is a real numbers. Then, there are following four possible cases in principle by combining two type matrices,

$$
\begin{aligned}
&\text{case I} && M_u = M_1 && M_d = M_1, \\
&\text{case II} && M_u = M_1 && M_d = M_2, \\
&\text{case III} && M_u = M_2 && M_d = M_1, \\
&\text{case IV} && M_u = M_2 && M_d = M_2.
\end{aligned}
$$

The case I is the well known Fritzsch ansätze[5]. Although this ansätze is successful for the V_{us} element as shown in Eq.(1), it fails for V_{cb} as far as $m_t \geq 100\text{GeV}$. On the other hand, the case IV which is the ansätze proposed by Branco et al.[8] is successful not only for the V_{us} element but also for the V_{cb} element. Although this ansätze overcomes the fault of the Fritzsch ansätze, it cannot reproduce the observed ratio of $|V_{ub}|/|V_{cb}| = 0.08 \pm 0.02$[9]. Actually, the case IV predicts the larger value than 0.12 for this ratio. Then we examine other two cases. First one is the case II;

$$
M_u = M_1 \quad , \quad M_d = M_2.
\tag{3}
$$

The matrices U_u and U_d are defined as the unitarity matrices which diagonalize the

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hermitian matrices $H_u = M_u$ and $H_d = M_d M_d^\dagger$, respectively,

$$U_u^\dagger H_u U_u = D_u \ , \qquad U_d^\dagger H_d U_d = D_d \ , \tag{4}$$

where $D_u = \text{diag.}(m_u, m_c, m_t)$ and $D_d = \text{diag.}(m_d^2, m_s^2, m_b^2)$. In the NNI basis, we can extract phases from each quark mass matrix by the use of the diagonal phase matrices. Since phases of the mass matrices can be isolated, we are able to write

$$U_u = \phi_u O_u \ , \qquad U_d = \phi_d O_d \tag{5}$$

where $\phi_u = \text{diag.}(e^{ip_u}, e^{iq_u}, 1)$, $\phi_d = \text{diag.}(e^{ip_d}, e^{iq_d}, 1)$ and O_u, O_d are orthogonal matrices. We define the phase matrix, $\Phi = \phi_u^* \phi_d = \text{diag.}(e^{ip}, e^{iq}, 1)$ with $p = p_d - p_u$ and $q = q_d - q_u$. Then the KM matrix is given by,

$$V_{KM} = U_u^\dagger U_d = O_u^T \Phi O_d \ . \tag{6}$$

In the case of Eq.(3), we obtain the KM matrix elements approximately,

$$V_{ud} \ \simeq \ \frac{1}{N_{ud}} \left(e^{ip} + 2^{-\frac{1}{4}} \sqrt{\frac{m_u m_d}{m_c m_s}} e^{iq} + 2^{-\frac{3}{4}} \sqrt{\frac{m_u m_d m_s}{m_t m_b^2}} \right), \tag{7}$$

$$V_{us} \ \simeq \ \frac{1}{N_{us}} \left(-2^{-\frac{1}{4}} \sqrt{\frac{m_d}{m_s}} e^{ip} + \sqrt{\frac{m_u}{m_c}} e^{iq} + \sqrt{\frac{m_u}{m_t} \frac{m_s}{m_b}} \right), \tag{8}$$

$$V_{ub} \ \simeq \ \frac{1}{N_{ub}} \left(2^{-\frac{1}{4}} \sqrt{\frac{m_d m_s}{m_b^2}} e^{ip} + \sqrt{\frac{m_u}{m_c} \frac{m_s}{m_b}} e^{iq} - \sqrt{\frac{m_u}{m_t}} \right), \tag{9}$$

$$V_{cd} \ \simeq \ \frac{1}{N_{cd}} \left(\sqrt{\frac{m_u}{m_c}} e^{ip} + 2^{-\frac{3}{4}} \sqrt{\frac{m_d}{m_s}} e^{iq} + 2^{-\frac{1}{4}} \sqrt{\frac{m_c m_d m_s}{m_t m_b^2}} \right), \tag{10}$$

$$V_{cs} \ \simeq \ \frac{1}{N_{cs}} \left(-2^{-\frac{1}{4}} \sqrt{\frac{m_u m_d}{m_c m_s}} e^{ip} + e^{iq} + \sqrt{\frac{m_c}{m_t} \frac{m_s}{m_b}} \right), \tag{11}$$

$$V_{cb} \ \simeq \ \frac{1}{N_{cb}} \left(2^{-\frac{1}{4}} \sqrt{\frac{m_u m_d m_s}{m_c m_b^2}} e^{ip} + \frac{m_s}{m_b} e^{iq} - \sqrt{\frac{m_c}{m_t}} \right), \tag{12}$$

$$V_{td} \ \simeq \ \frac{1}{N_{td}} \left(\frac{m_c}{m_t} \sqrt{\frac{m_u}{m_t}} e^{ip} + 2^{-\frac{1}{4}} \sqrt{\frac{m_d m_c}{m_s m_t}} e^{iq} - 2^{-\frac{3}{4}} \sqrt{\frac{m_d m_s}{m_b^2}} \right), \tag{13}$$

$$V_{ts} \ \simeq \ \frac{1}{N_{ts}} \left(-2^{-\frac{1}{4}} \frac{m_c}{m_t} \sqrt{\frac{m_u m_d}{m_t m_s}} e^{ip} + \sqrt{\frac{m_c}{m_t}} e^{iq} - \frac{m_s}{m_b} \right), \tag{14}$$

$$V_{tb} \ \simeq \ \frac{1}{N_{tb}} \left(2^{-\frac{1}{4}} \frac{m_c}{m_t} \sqrt{\frac{m_u m_d m_s}{m_t m_b^2}} e^{ip} + \sqrt{\frac{m_c}{m_t} \frac{m_s}{m_b}} e^{iq} + 1 \right), \tag{15}$$

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where $1/N_{ij}$s are normalization factors.

In order to estimate the absolute values of KM matrix elements $|V_{ij}|$, we must know the values of the masses of six quarks on the same energy scale. Following the study by Koide[10], we obtain the values of quark masses at 1GeV by using the 2-loop renormalization group equations,

$$m_u = 0.0056 \pm 0.011 \ , \ m_d = 0.0099 \pm 0.0011 \ , \ m_s = 0.199 \pm 0.033 \ ,$$

$$m_c = 1.316 \pm 0.024 \ , \ m_b = 5.934 \pm 0.101 \ , \ m_t = 349.5 \pm 27.9 \ (\text{GeV}) \ ,$$

where $\Lambda_{\overline{MS}}^{(5)} = 0.195$GeV. Hereafter, we use the central values for the numerical estimation. If we put the phase parameters on $p = -73°$ and $q = 47°$, we obtain the absolute values of the KM matrix elements;

$$|V_{KM}| = \begin{pmatrix} 0.97556 & 0.21971 & 0.00455 \\ 0.21957 & 0.97452 & 0.04580 \\ 0.00917 & 0.04510 & 0.99894 \end{pmatrix}. \tag{16}$$

All of these nine values are consistent with the experimental data[9] as;

$$|V_{KM}| = \begin{pmatrix} 0.9745 \text{ to } 0.9757 & 0.219 \text{ to } 0.224 & 0.002 \text{ to } 0.005 \\ 0.218 \text{ to } 0.224 & 0.9736 \text{ to } 0.9750 & 0.036 \text{ to } 0.046 \\ 0.004 \text{ to } 0.014 & 0.034 \text{ to } 0.046 & 0.9989 \text{ to } 0,9993 \end{pmatrix}. \tag{17}$$

The unitarity triangle obtained from the KM matrix elements of the Eq.(16) is shown in Fig.1. Here, in order to describe the experimentally allowed region, we used the recent JLQCD result[11] of the Lattice Calculation for the theoretical parameters $\hat{B}_K$ and f_{B_d} as follows:

$$\hat{B}_K = 0.76 \pm 0.04 \ , \qquad f_{B_d} = 0.19 \pm 0.01 \text{GeV} \ . \tag{18}$$

The position of the vertex point of this unitarity triangle is on the first quadrant of the $\rho - \eta$ plane. So the case II is an ansätze which put the unitarity triangle on the first quadrant.

Next, we consider the case III;

$$M_u = M_2 \quad , \quad M_d = M_1. \tag{19}$$

The KM matrix elements are obtained analogously. For example, V_{cb} is given as,

$$V_{cb} = -2^{-\frac{1}{4}} \frac{m_s}{m_b} \sqrt{\frac{m_u m_d}{m_c m_b}} e^{ip} + \sqrt{\frac{m_s}{m_b}} e^{iq} - \frac{m_c}{m_t}. \tag{20}$$

Unfortunately, we cannot reproduce the experimental value of $|V_{cb}|$ even if any values of the phase parameters are taken.

In this paper, we examine the unitarity triangle on the first quadrant of the $\rho-\eta$ plane by using the quark mass matrices in the NNI basis. Assuming $(M_u)_{12} = (M_u)_{21}^*$ and $(M_d)_{12} = (M_d)_{21}^*$, four cases(I~IV) are considered. There is only one case, $i.e.$ case II, which can reproduce all absolute values of the KM matrix elements. The quark mass matrix combination of the case II implies the unitarity triangle with the vertex on the first quadrant. If the B-factory experiments at KEK and SLAC will restrict the experimentally allowed region of the unitarity triangle in the first quadrant of the $\rho - \eta$ plane, our proposed simple model can be a candidate for the quark mass matrices.

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Figure Captions

Fig.1 : The unitarity triangle when the phase parameters are put on $p = -73°$ and $q = 47°$ in the case II.